Phase Mapping of Cardiac Fibrillation

Karthikeyan Umapathy, PhD; Krishnakumar Nair, MD; Stephane Masse, MA; Sridhar Krishnan, PhD; Jack Rogers, PhD; Martyn P. Nash, PhD; Kumaraswamy Nanthakumar, MD

Phase is a descriptor that tracks the progression of a defined region of myocardium through the action potential and has been demonstrated to be an effective parameter in analyzing spatiotemporal changes during fibrillation. In this review, the basic principles behind phase mapping are presented mainly in the context of ventricular fibrillation (VF), atrial fibrillation (AF), and fibrillation from experimental monolayer data. During fibrillation, the phase distribution changes over time, depending on activation patterns. Analyzing these phase patterns provides insight into the fibrillatory dynamics and helps clarify the mechanisms of cardiac fibrillation and modulation by interventions. Winfree introduced the phase analysis to study cardiac fibrillation in the late eighties. This time-encoding technique deals with a scenario where the activation periods are the same over the surface being mapped. To deal with the scenario of varying activation period over the mapped surface (common in animal and human fibrillation models), Gray et al introduced the state-space encoding concept from nonlinear dynamics.

In analyzing spatiotemporal phase maps constructed from electric or optical mapping of the surface of heart during VF, points around which the phase progresses through a complete cycle from $-\pi$ to $+\pi$ are of great interest. At these points, the phase becomes indeterminate and the activation wave fronts hinge to these points and rotate around them in an organized fashion. These points in the phase map are called phase singularity (PS) points. Bray et al developed a procedure to locate PS points in a phase map. Nash et al used phase mapping to study the entire ventricular epicardium of human hearts with a sock containing 256 unipolar contact electrodes. The development of this phase mapping tool has led to better understanding of fibrillation dynamics as evidenced by the use of phase mapping in detecting PS and their role in demonstrating organization during VF. Some of these works and their findings are (1) PS colocalize with anatomic heterogeneities, and their spatial meandering is modulated by these heterogeneities, (2) PS correlates with the locations of wave breaks, (3) myopathic human hearts, phase maps were used to show that the organization of electric activity were characterized by wave fronts emanating from a few rotors, and (4) phase mapping technique has also been applied to investigate the mechanism of fibrillation.

Clinical electrophysiologists innovating therapies for both AF and VF are commonly not aware of phase mapping. This review addresses this shortcoming with the hope that by introducing the basic concepts of phase mapping to a greater audience, there will be an opportunity to devise therapies for these arrhythmias on a mechanistic basis.

Methodological Considerations

Amplitude, frequency, and phase are 3 important parameters that characterize signals such as electrograms obtained during cardiac mapping. With relevance to this article, the signal of interest is the electrogram that is acquired either by electric or optical mapping of the cardiac tissue. Understanding the fundamental signal characteristics, especially the concept of phase and its technical details, are of importance in generating and understanding phase mapping.

Data observed over a time period can be generalized as a time series. When a time series contains patterns at regular intervals of time, it is said to be periodic, and aperiodic otherwise. In signal processing terminology, the variation of samples (of some physical quantity such as voltage or temperature) over time or space is defined as a signal. A signal can be characterized by 3 parameters: (1) amplitude, the magnitude of the physical quantity measured over a particular time interval, (2) frequency, the fundamental repetition cycle of the numeric values, and (3) phase, the different stages within 1 cycle of a signal divided into $360^\circ$ or $2\pi$ radians. Supplemental Figure 1 illustrates an example of a sinusoidal signal, a surface ECG, and an endocardial intracardiac electrogram. Phase can be better visualized from a rotating phasor diagram as shown in supplemental Figure 2. In supplemental Figure 2A, a rotating vector (blue arrow) is shown on a complex plane at different stages around a unit circle and the effective influence of this vector on each of the axes is plotted as a signal (sine and cosine). The 2 signals shown in supplemental Figure 2A are time-delayed versions of each other, or in other words, signals that are phase-shifted by
90° (corresponding to the angle between the axes). The position of the vector on the circle can either be given by the length of the vector “r” and angle “θ” in polar coordinates or “a + j b” in Cartesian coordinates. The “j” represents the imaginary part of the complex number. At any given time the position of the vector with respect to the origin gives the instantaneous phase of the signal and is represented by “θ” in polar coordinates or given by “tan⁻¹ (b/a)” in Cartesian coordinates. The number of times the vector rotates around the circle per unit time is proportional to the angular frequency. Depending on the position of the rotating vector, the amplitude of the signal varies over time. If the magnitude of the vector and the angular velocity with which it rotates are unchanged, the resulting signal is a sinusoid. By changing these 2 parameters, a variety of real world signals can be reproduced.

In supplemental Figure 2B, 2 phasors are shown with a delay (“θ_d”) in their positions, the corresponding 2 signals are plotted, and the delay is shown as a constant phase difference between the signals. This can be visualized in cardiac electrophysiology terms as an activation wave front that travels over the surface of the heart, and 3 electrodes, E1, E2, and E3, are in its path. Supplemental Figure 2C shows the 3 electrodes, E1, E2, and E3, and the right side of the panel shows 3 electrograms (sinusoids for simplicity). The observable phase difference between the signals is proportional to the distance between the electrodes and the velocity of the traveling activation front.

Another convenient way of describing a signal is using 2 counter-rotating vectors instead of a single rotating vector as shown in supplemental Figure 2D. By introducing counter-rotating functions, we now have both positive and negative frequency components, that is, 2 vectors rotating in opposite directions with the same angular velocity can be seen as 2 vectors rotating at “ω” and “−ω” angular frequencies. Adding these 2 exponential functions results in a projection on the real axis, yielding the real component of a signal (or a real signal) and subtracting these 2 functions results in the imaginary component of the signal. By this approach, we could represent any real signal as a sum of equal positive and negative frequency components. In simple words, we need both positive and negative frequency components to represent a real signal. If for any reason we eliminate or reduce either the positive or negative frequencies, it will result in a complex signal of the form “r/θ” or “a + j b.”

Although the above discussions present the phase parameter using an example of a sinusoid, which is periodic (or fixed cycle length), it must be noted that during cardiac fibrillation the electrograms are not periodic and exhibit time-varying cycle lengths. In this case, the phase measured as an absolute “time delay” from a reference at 2 spatially separated electrode locations with corresponding electrograms having different cycle lengths is of no relevance. In other words, time-encoding of phase in not reliable in studying cardiac fibrillation, and an alternate approach is needed where each cycle is coded to 0 to 2π radians irrespective of its cycle length. Hence, for studying the spatial distribution of phase during cardiac fibrillation, a 2-variable state-space approach is used. In the state-space encoding approach, the scalar electrograms recorded at an electrode location during cardiac fibrillation is treated as an output of a nonlinear dynamical system. If the nonlinear dynamic system is characterized by a finite number of states,
the transition among these states over time can be represented using a multidimensional state-space and the trajectory over all the states can be used to identify organization or determinism of the dynamic system. More details on this approach of computing phase are provided in a later section, “Phase Analysis.”

Intracardiac Electrograms, Data Acquisition, and 2D Maps

Before elaborating the details of the phase analysis and construction of phase maps, it is essential to briefly revisit the acquisition techniques that are used in most of the existing studies and relate them to the temporal and spatial components of the VF dynamics. Electric and optical mapping are 2 well-known methods that are commonly used for acquiring electrograms during fibrillation. Because the material that will be covered in this article is common to both electric and optical mapping approaches, we restrict our presentation to the electric mapping approach only.

In the electric mapping approach, the surface of the heart (epi and/or endo) is brought in contact with an array of unipolar and/or bipolar electrodes with necessary acquisition hardware and analyzed using specific software. Figure 1A shows an image of the heart in a Langendorff setup with an epicardial sock electrode array. The electrode array is made up of a grid of electrodes geometrically distributed to cover the surface of the epicardium. A sample portion of the electrode grid is shown in Figure 1B. During acquisition, each of these electrodes records an averaged local activity (for unipolar) of the heart for each sampling instant of the acquisition. Figure 1C shows a sample portion of the electrode grid at different time instants, t1, t2, and t3. Sampling over time gives the temporal evolution of the local electric activity or simply an electrogram at a particular location on the surface of the heart. As an example, an intracardiac electrogram is superimposed over the grids to show its relation with time and space. For each instant of time, we can ascertain the spatial distribution of electrograms over all the electrodes in the grid.

Although mapped with several electrodes in an array, the spatial resolution (number of electrodes per unit area) is usually inadequate to represent every single point on the surface of the heart with detail. This is not the case with optical mapping because high spatial resolution (in the order of micrometers) can be achieved. However, clinical usage of optical mapping in humans may not be available in the immediate future, owing to the toxicity of the required voltage-sensitive dyes. To overcome this limitation of electric mapping, data interpolation is often performed to increase the spatial resolution of the electrogram data. Using this method, the surface area between the locations of the electrodes is filled with values interpolated from the electrode data that surround this area. Figure 2 illustrates the generation of interpolated electrograms and an example 2D voltage map constructed from a VF episode using different pseudospatial resolutions. If this is done for every instant of time and
viewed dynamically (e.g., as a movie), a spatiotemporal voltage map is obtained. Similarly, frequency maps can be constructed from the electrograms, widely known as dominant frequency (DF) maps. Because the conventional computation of frequency is done over a time segment, the DF mapping currently used in studying fibrillation, may it be AF or VF, only produces time-averaged frequency information over space. This means the conventional DF techniques currently used mainly capture spatial information in an arrhythmia that has significant temporal evolution. If, instead, spatiotemporal frequency maps are to be constructed, then combined time-frequency methods should be explored. Nevertheless, generating and analyzing these spatiotemporal maps are very helpful in assimilating the spatial and temporal changes during fibrillation and tracking the activation patterns dynamically. Phase analysis becomes a valuable tool in this regard in detailing both spatial and temporal information when appropriately used.

**Phase Analysis**

Phase analysis of the electrograms is one method of identifying and quantifying spatiotemporal organization of VF. Because we are interested in tracking the activation patterns both spatially and temporally during cardiac fibrillation, a phase parameter would be more suited because it is not directly dependent on the actual amplitude of the electrograms. The electrogram amplitude information during fibrillation is often ambiguous, prone to acquisition artifacts, and highly influenced by the physiological and anatomic substrate, hence it is not a preferred measure to study the spatiotemporal activation patterns. Frequency is also an amplitude-independent measure and could provide information on the spatial organization of VF but has limitations in tracking the temporal variations as explained in the previous section.

**Temporal Organization**

If cardiac fibrillation is not random and does show temporal organization, then it is possible that an intersample temporal correlation (at least in short time segments) exists in the electrogram. In other words, it should be possible to show that the future time samples in a fibrillation electrogram are dependent or related to the present and past time samples.

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**Figure 3.** Phase-space plots for 3 different scenarios: A, sinusoid; B, sample VF segment; and C, random noise. Subplots D and E illustrate the phase computation using phase-space plots of a sinusoid and a VF segment.
This relation can be graphically visualized using phase-space 2D plots (also known as return maps). The concept for phase-space plots is taken from state-space mapping of nonlinear dynamics and applied to cardiac electrophysiology by treating fibrillation as an output of a nonlinear dynamical system. In a phase-space plot, we plot the signal \([V(t), a \text{ VF electrogram}]\) and its time delayed version \([V(t−τ)]\) on the \(X\) and \(Y\) axes, respectively. If there is a consistent relation between the present and past time samples, the trajectory of the curve forms a definite shape and loops around a central point called the attractor. If it is a random relation, then the trajectory has no definite shape and spreads all over the space. This trajectory can be seen as a transition between the states of a dynamical system, and, if there is determinism in these transitions, then the shape of trajectory has definitive shape. The choice of \(τ\) is critical and typically chosen to be 25% of the VF cycle length. Figure 3A through C shows phase-space plots of a periodic sinusoidal wave, a VF electrogram, and a random sequence of numbers. We see that the trajectory of the sinusoidal wave is condensed and occupies a very small overall area in the phase-space plot. In contrast, the trajectory of the fibrillation electrograms is dispersed but loops around a central point, indicating some degree of temporal organization. In the case of the random sequence, the trajectory is all over the space indicating disorganization. From the phase-space plot, the phase at each instant in time (instantaneous phase) or at each point in the trajectory can then be computed using the 2 variables, \(V(t)\) and \(V(t−τ)\) as long as the central point is at the origin (or a stable center like the mean of \(V(t)\) that does not vary from cycle to cycle\(^{11}\)).}

**Hilbert Transform**

In the previous section, we explained the phase-space plots and the extraction of instantaneous phase of the electrograms. However, this approach of computing phase requires a judicial choice of \(τ\). To overcome this dilemma, the Hilbert Transform (HT)-based approach was used by some of the recent studies in computing the instantaneous phase.\(^4,5\) In simple terms, HT is another way of generating a phase-shifted signal from the original signal and using these 2 signals the instantaneous phase is computed. For interested readers, more details are provided in the methods supplemental section. The HT approach in computing the instantaneous phase of a signal is a robust approach; however, the signal must meet some conditions (refer to Limitations section) for reliable phase computation.\(^5\)

**Spatial Organization**

To identify the presence of spatial organization, we need to analyze the phase of all the electrograms that are acquired over the epi or endocardium at every instant of time. During VF, activations propagate along the reentrant path or the wave fronts rotate around a point or an area on the surface of the heart. These rotating wave fronts around these points or areas will induce a progressive increase in phase over space covering a complete cycle of phase from \(-π\) to \(+π\). When this happens, a “phase singularity (PS)” is observed at the central point. PS points are important in the study of fibril-
lation dynamics because these points are considered to be the pivot points of reentrant circuits. These points might indicate a structural or a functional anomaly that initiates curved cardiac excitation waves. In spiral wave reentries, these points form the core of the unexcited but excitable cardiac tissue or tip of a 3D spiral wave. Identifying these points might lend to the possibilities of ablation strategies that would modulate VF. Figure 4 illustrates this rotating phase concept. Activation patterns when blocked by anatomic or physiological heterogeneities can rotate around the heterogeneities, as shown in Figure 4. The area around the rotating axes of the wave front then exhibits progressive phase changes along the path of the rotation, as shown in Figure 4. Now if we compute instantaneous phase over the surface of the heart during VF using the state-space or HT methods explained earlier and construct a dynamic 2D map over time, we should observe rotating phase patterns along the reentrant paths. Because phase varies from \(-\pi\) to \(\pi\), and by color-coding this range from blue to red, we obtain dynamic 2D maps as shown in Figure 5. Figure 5A shows a surface electrode array; Figure 5B shows the phase map constructed from human VF data. The electrode array is superimposed on the phase map for a visual correlation of actual electrode locations with the phase map. Figure 5D through 5G shows the complete rotating cycle of the phase pattern in 4 snapshots (t1 to t4) over time. The curved arrow marks in the bottom panel indicate the direction of rotation about a phase singularity. Figure 5C shows the electrograms recorded at locations 1 through 5 that are marked in Figure 5D, indicating the locations of the real electrodes (ie, not interpolated) along the path of the rotating phase pattern. (Either a black or a white letter is used on the phase maps for better visibility.) A clear phase progression can be observed in the electrograms and is indicated by the pink line. It is important to recognize here that the pink line does not indicate a single time instant that matches the phase snapshots. The alphabetic markings on the phase maps are provided only for the geometric correlation with the phase progression in the timing diagram. Depending on the life of the PS point (or) in other words as long as the physiological conditions are not disturbed by internal or external mechanisms, the phase pattern in the phase maps and the corresponding phase delays in the electrograms repeat cyclically giving rise to rotors. A rotor is identified if a PS persists long enough for the phase pattern to cycle at least 2 times around the PS.
Applications in the Study of Human VF, AF, and Experimental Monolayer Models

In this report, we present some of our phase analysis of fibrillation data demonstrating the identification of different types of reentrant circuits and the relation of phase maps with frequency maps. Figure 6A and 6B shows a phase-space plot and the DF map of a human VF episode. From Figure 6A, the organization within the VF episode is evident. Figure 6B shows the spatial distribution of the dominant frequency of the VF episode, and a high-frequency area approximately between the 4 to 7 o’clock positions can be observed. Two instances of the corresponding phase map are shown in the bottom panel of Figure 6D and 6E. Interestingly, in this VF episode we observed a migratory rotor. The area of the migration is shown using a dotted black line in both the phase map snapshots, and this area colocalized the high-frequency area observed in the DF map in Figure 6B. Figure 6F gives a detailed picture of the migratory path of the rotor. The black dotted line shows the area of migration, the purple line shows the migratory path of the rotor, the small red circle shows the sense of rotation of the rotor, and the green arms are representative of the rotating wave fronts. The rotation of the rotor and the movement along the migratory path were of opposite directions, and we also observed the migratory path approximately circumscribing the high-frequency area observed in Figure 6B. This observation was correlated with scar analysis using USG images and bipolar electrogram amplitudes measured at these locations on the heart. The boundaries of the high-frequency area and the migratory area of the rotor colocalized to the border zone of scar/healthy tissue. We verified the phase pattern by analyzing the electrograms along the migratory path of the rotor, as shown at locations 1 through 7 on the phase maps. The electrograms are shown in Figure 6C, with evidence of phase progression seen, validating the presence of a PS and a rotor. A movie file is provided in the Data Supplement as a demonstration of the migratory rotor. These findings have clinical implications for therapies: a fixed stable rotor and thus a pointed ablative strategy is unlikely to modulate fibrillation, rather an ablative strategy with a wide area that encloses the area of migration of the rotor may have better success and needs to be tested. Indeed, clinical ablation of atrial fibrillation has moved to a wider area of ablation, and this may have mechanistic implications to the area of rotor migration in addition to other mechanisms. However, these concepts must be tested in the clinical setting.
The top panel of Figure 7A through 7C shows the phase map of “figure 8” reentry in a humanVF episode. The geometry of the mapped area in this example is different because it is from the right ventricle, including right ventricular outflow tract. Curved arrows indicate the sense of rotations. Figure 7D shows the electrograms around the 2 counter-rotating phase patterns, and the phase progression is evident from the plot. Our previous studies have demonstrated “figure 8” reentry on the epicardium using limited epicardial mapping; the global mapping strategy with phase mapping provides a greater perspective of the “figure 8” reentry in the context of a rotor.

In the bottom panel, Figure 7E through 7G shows 3 instances of phase maps constructed using a humanVF segment. These phase maps show a migrating rotor. Curved arrows show the direction of rotation. Figure 7H shows the 3D trajectory of the migratory rotor superimposed with the surface electrode array at the top and bottom of the cube. X and Y axes show the spatial coordinates and the Z axis the time.

The top panel of Figure 8 shows 4 time snapshots of the phase map of “figure 8” reentry from an AF episode of a dog. The acquisition was made using a plaque electrode array consisting of 7×8 electrodes, hence the geometry of the phase map is almost square. From the figure (A through D), the rotating phase pattern is evident, indicating the presence of a reentrant circuit and the spatiotemporal organization of electric activity. To date, mechanisms that trigger have been characterized well in AF;
However, in humans, reentry that sustains AF is poorly characterized; using phase mapping will provide better understanding of reentry in AF in humans and thus provide mechanistic basis for ablation strategy such as region of ablation, lines of ablation, and fractionation of electrograms.

Considering the long-term goal of arriving at preventive and therapeutic solutions to avert sudden cardiac deaths, it is also essential to study the arrhythmias at a cellular level. Phase mapping can be applied to the controlled 2D cardiomyocyte preparations as well to study their fibrillatory behavior. We grew murine HL-1 cardiomyocytes in monolayer preparation, and the spontaneous fibrillatory activities of these 2D preparations were mapped using an optical mapping system. A surface area of 6 to 10 mm² was mapped and pseudoelectrograms were extracted from the optical data. Phase maps were then constructed using these pseudoelectrograms. The bottom panel of Figure 8E through 8F shows 2 time snapshots of the phase map computed using 1 of our monolayer experimental models. The curved arrow shows the direction of rotation and the straight arrow shows the location of a wave break. We have been able to use these techniques in studying the basic mechanistic concepts as they relate to bipolar electrograms, CFAE, rotor location, and wave breaks. The Table provides a summary of our research findings using phase maps to study cardiac fibrillation in human, animal, and experimental monolayer models.

**Limitations**

We have presented a practical yet detailed methodology behind the phase map construction. Although the phase map provides valuable insight into spatiotemporal organization during fibrillation, a few practical limitations should be kept in mind for a meaningful phase map interpretation. The unipolar electrograms are created by integrating the electric potential over a small volume of adjacent cardiac tissue. Hence, they may not be reliable for studying highly localized activation patterns. Especially in areas of scar and other anatomic anomalies, the unipolar electrograms are often due to far fields, and this may bias the spatial distribution of phase. Phase is also misleading for slow extracellular electrograms in which T-waves are present. Due to the presence of T-waves, there are 2 phase cycles for each cardiac cycle. This is not as much of an issue in tachyarrhythmias in which T-waves merge with following QRS. The number of electrodes used to map the entire epicardium or endocardium of the heart is often inadequate. Data interpolation is used to fill in the spaces between the electrodes by interpolated electrograms. The quality of the interpolated electrograms and their reliability depends on the proximity of the actual electrodes to
Table. Research Findings Using Phase Maps in Studying Cardiac Fibrillation

<table>
<thead>
<tr>
<th>No.</th>
<th>Research Findings Using Phase Maps</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Demonstrated the spatiotemporal organizational aspects of human VF and that the organizational centers are unstable and migratory in nature.</td>
<td>Characterization of human VF mechanism</td>
</tr>
<tr>
<td>2</td>
<td>The interface of scar-healthy tissue forms a favorable substrate for the rotors to hinge and migrate along the border.</td>
<td>Insight on ablation strategies</td>
</tr>
<tr>
<td>3</td>
<td>Established correlations between the genesis of fractionated electrograms with the certain conditions of rotor formation and wave break.</td>
<td>Insight on electrogram guided AF ablation strategies</td>
</tr>
<tr>
<td>4</td>
<td>Presence of 3D scroll waves (a single rotating filament through the myocardium) demonstrated using needle electrodes in human VF.</td>
<td>Insight on the participation of myocardium</td>
</tr>
<tr>
<td>5</td>
<td>There is a marked difference between the organizational aspects of epicardium and endocardium.</td>
<td>Characterization of human VF mechanism</td>
</tr>
<tr>
<td>6</td>
<td>Wave fronts measured using optical mapping during human VF can be large (&gt;8 cm long), suggesting that there is a limited number of wave fronts during VF.</td>
<td>Insight on VF organization and dynamics</td>
</tr>
</tbody>
</table>

the location of the interpolated electrograms. The larger the distance between the true electrodes, the poorer the interpolated electrograms, or we might miss some information. The reliability of phase computations depends on how well the activation detection is performed. It is essential to extract the cycle end points for each of the activations during fibrillation so that phase can be computed without ambiguity. Techniques that use the HT approach must ensure that the mean potential over each activation cycle is subtracted from the original electrogram for reliable phase computations.

Although phase mapping is a promising technique in analyzing fibrillation dynamics, it requires simultaneous mapping of a significant portion of epicardium or endocardium. However, this limitation is overcome to a certain extent by the innovative catheter technologies, whereby simultaneous multielectrode acquisitions are possible. Examples are Constellation Catheter (Boston Scientific) and Basket Catheter (EnSite).

Because the main objective of this report is to present phase mapping to clinicians who are mapping endocardium, epicardium, or both, we restricted our presentation of phase maps only on 2D surfaces (ie, epicardial or endocardial); however, in the study of VF fibrillation dynamics, it is important to analyze and take into account 3D wave front behavior across epicardial, intramural, and endocardial regions.

Conclusion

A higher level of quantification of cardiac fibrillation is needed in clinical electrophysiology to define mechanisms of cardiac fibrillation in humans. Analyzing the complex spatiotemporal patterns seen during fibrillation on the surface of the heart can be greatly simplified by identifying and analyzing phase singularities. The observation of phase singularities has led to the quantification of fibrillation in terms of rotor number and lifespan, wave break, and to the elucidation of the mechanisms underlying the formation and termination of rotors. Further studies are needed to investigate these findings in more detail and to determine the conditions that modulate this behavior. These include increasing or decreasing mobility of reentrant sources and increasing or decreasing breakup and stability of rotors. The outcome of such studies would have clinically important implications in providing a step toward defining targets for mechanism based therapies in patients whose lives are affected by cardiac fibrillation.

**Sources of Funding**

This study was supported by Canadian Institutes of Health Research grant NA 777687 to Dr Nanthakumar.

**Disclosures**

None.

**References**


**Key Words:** phase maps • phase-space • phase singularity • rotors • wave break • cardiac fibrillation
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Circ Arrhythm Electrophysiol. 2010;3:105-114
doi: 10.1161/CIRCEP.110.853804

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SUPPLEMENTAL MATERIAL
Methods - Supplement

Hilbert Transform (HT)

Recall the discussion in the “Introduction” section on describing signals using two counter rotating phasors. Using this approach, we explained that every real signal should have both positive and negative frequencies and if one of them is eliminated then the signal becomes complex of the form "a + j b" i.e. the imaginary component is non-zero. The HT can be used to transform a real signal to a complex signal by removing the negative frequencies in a signal. Once this is done we have the signal in the complex form "a + j b" which is in an advantageous form to compute the instantaneous phase using "tan^-1(b/a)". In simple words the HT introduces -90 degree phase shift to the positive frequencies and +90 for the negative frequencies of a signal and when this phase shifted signal is added as imaginary part with the original real signal the negative frequencies cancel out\(^{15}\). Although a simple but robust approach, the signal has to meet a few initial conditions for proper phase computation using HT\(^9\). For interested readers, HT can be easily performed using MATLAB, a computing software from MATHWORKS Inc. We also give below the equation for HT:

Let \( x(t) \) be the signal under study and the analytical version of the signal is given by

\[
z(t) = x(t) + jH[x(t)]
\]

Where \( H \) is the Hilbert transform operator and is given by
\[ H[x(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \]

The complex analytical signal \( z(t) \) can then be written in polar coordinates as
\[ A(t) e^{\phi(t)} \] with \( A(t) \) and \( e^{\phi(t)} \) being the instantaneous amplitude and phase of the signal.

**SUPPLEMENTAL FIGURES IN NEXT 2 PAGES**
Supplemental Figure 1

Data Points: 0, 0.31, 0.59, 0.81, 0.95, 1, 0.95, 0.81, 0.59, 0.31, ...

Intracardiac Electrogram during VF
Surface ECG during VF
Supplemental figure 2

(a) \[ \mathcal{Z} = \cos(\omega \theta) - \sin(\omega \theta) \theta \]

(b) \[ \mathcal{Z} = \cos(\omega \theta) + \sin(\omega \theta) \theta \]

(c) Phase difference

(d) Green arrow

(e) Pink arrow

(f) Phase difference

(g) Green arrow

(h) Pink arrow

(i) Phase difference

(j) Green arrow

(k) Pink arrow

(l) Phase difference

Supplemental figure 2
Figure Legends:

Supplemental figure 1: A sample time-series of sinusoidal data and the plots of a sinusoid, surface ECG, and intracardiac electrogram.

Supplemental figure 2: (a) A rotating vector around a unit circle and the projection of the vector on imaginary and real axis as sine and cosine signals. (b) Phase difference using the phasor diagram with two vectors rotating with a constant delay between them. (c) Phase difference using electrodes that are separated spatially and encounter a traveling wavefront. (d) Demonstration of signal representation using two counter rotating phasors.

Video Legends:

Video1: Using phase map the movie demonstrates a migratory rotor during human VF that reveals the hidden spatial organization.